Preprint #1
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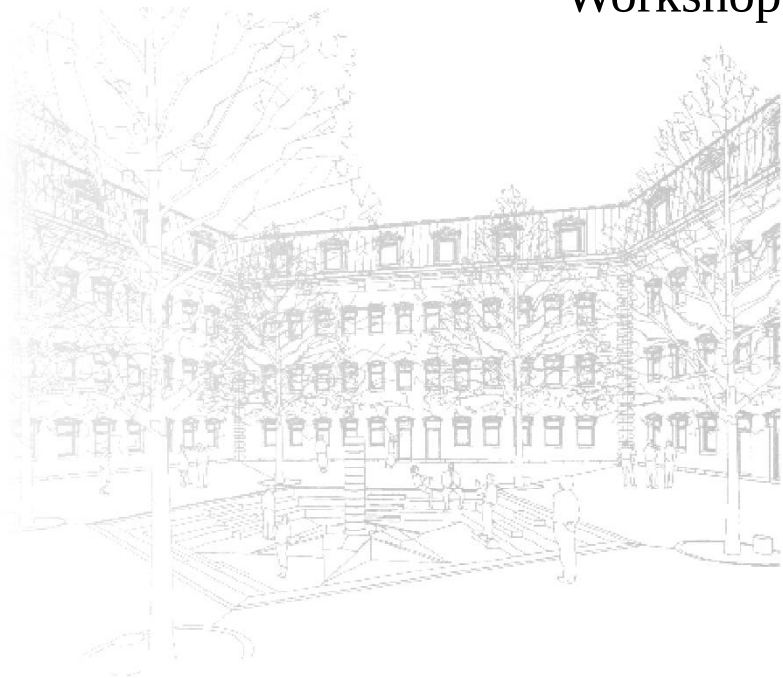
Maurice Boffa's 60th Birthday Workshop



Institut de Mathématique et d'Informatique
Université de Mons-Hainaut
Phone: +32 65 37 35 07 — Fax: +32 65 37 33 18
Web: http://www.umh.ac.be/math/institut

Maurice Boffa’s 60th Birthday Workshop

Programme

Thursday March 23, ULB Local 2NO906

14:30 Georges HANSOUL (Liège)
Décoration booléenne de graphes

15:15 Véronique BRUYÈRE (Mons)
Automates et systèmes de numération

16:00 Roland HINNION (Bruxelles)
Tree-properties for ordered sets

16:40 Coffee break

17:00 Françoise POINT (Mons)
Groupes satisfaisant une identité et propriété de Milnor

17:40 Olivier ESSER (Bruxelles)
The consistency strength of a positive set-theory

18:20 Christophe TROESTLER (Mons)
La récursivité vectorielle est-elle plus forte que la récursivité scalaire ?

$\sim$19:30 Dinner

$\sim$22:00 Transfer to Mons

Friday March 24, UMH Local 0A11 (Pentagone)

9:00 Alexis BES (Paris)
La conjecture d'Erdos-Woods

9:40 Arnaud MAES (Mons)
Entrelacs Brunniens et généralisations d'après Stanford

10:20 Jan DENEF (Leuven)
Macintyre's elimination theorem and p-adic groups

11:20 Coffee break

11:40 Marcel CRABBE (Louvain-la-Neuve)
L'axiome de l'infini dans NFU

12:20 André PETRY (Liège)
Faire de l'Analyse non standard sans Logique ?

13:00 Lunch

16:00 Angus Macintyre will receive the Honorary Degree of Doctor honoris causa of the Faculty of Sciences of the University of Mons-Hainaut.

Saturday March 25, UMH Grands Amphithéâtres

9:45 Welcome

10:10 Dirk VAN DALEN (Utrecht)
Foundations of Brouwer's Intuitionism

11:00 Coffee break

11:30 Serge GRIGORIEFF (Paris)
Automates et mots transfinis

12:20 Lunch

13:50 Ulrich FELGNER (Tübingen)
Fonctions régressives

14:40 Henk BARENDREGT (Nijmegen)
Set theory versus type theory as foundation

15:30 Coffee break

16:00 Marco FORTI (Pisa)
Wanted: a strongly comprehensive theory of collections

16:50 Angus MACINTYRE (Edinburgh)
Various exponentials occurring in algebra and number theory, and their model theory

17:40 Elisabeth BOUSCAREN (Paris)
Théorie des modèles et Conjecture de Manin-Mumford (d'après Ehud Hrushovski)

20:00 Conference Dinner

List of participants

Henk Barendregt (Nijmegen)
Chantal Berline (Paris)
Alexis Bès (Paris)
Maurice Boffa (Mons)
Elisabeth Bouscaren (Paris)
Véronique Bruyère (Mons)
Zoé Chatzidakis (Paris)
Raf Cluckers (Leuven)
Marcel Crabbé (Louvain-la-Neuve)
Dirk van Dalen (utrecht)
Jan Denef (Leuven)
Jean Drabbe (Bruxelles)
Gregory Duby (Bruxelles)
Daniel Dzierzgowski (Trasys)
Olivier Esser (Bruxelles)
Ulrich Felgner (Tübingen)
Thomas Forster (Cambridge)
Marco Forti (Pisa)
Jean-Sylvestre Gakwaya (Mons)
Paul Gochet (Liège)
Serge Grigorieff (Paris)
Georges Hansoul (Liège)
Dirk van Heule (Bruxelles)
Pol Henrard (Louvain-la-Neuve)
Roland Hinnion (Bruxelles)
Albert Hoogewijs (Gent)
Pol Lambert (Diepenbeek)
Tierry Libert (Bruxelles)
Angus Macintyre (Edinburgh)
Arnaud Maes (Mons)

Christian Michaux (Mons)
Didier Misercque (Bruxelles)
André Pétry (Liège)
Françoise Point (Mons)
Paul van Praag (Mons)
Armin Rigo (Bruxelles)
Jean Roger Roisin (Louvain-la- Neuve)
Christophe Troestler (Mons)
Patricia Wantiez (Mons)
Karim Zahidi (Gent)

1 Georges HANSOUL (Liège)

Décoration booléenne de graphes

Un espace est dit pseudo-indécomposable s'il n'est pas somme de deux espaces, tous deux non homéomorphes à lui. Il est dit primitif s'il est compact, métrique et admet une base d'ouverts fermés pseudo-indécomposables. Des exemples sont donnés par l'espace triadique de Cantor, les ordinaux dénombrables non limites et les algèbres de Lindenbaum de nombreuses théories usuelles. Les espaces primitifs peuvent être étudiés par le biais de décorations, id. d'une fonction d définie sur un graphe et à valeur dans les ouverts-fermés de l'espace de façon à ce que $d(x)$ soit déterminé canoniquement par les $d(y)$, où (x, y) est une arête du graphe. Comme pour les ensembles, on peut donner une construction de décorations de graphes non bien-fondés par approximations successives.

2 Véronique BRUYÈRE (Mons)

Automates et systèmes de numération

Le théorème de Cobham dit qu'un ensemble d'entiers reconnaissable par automate fini dans deux bases multiplicativement indépendantes est nécessairement une union de progressions arithmétiques. Depuis 1985, année où j'ai étudié ce théorème dans mon mémoire de fin d'études, beaucoup de travaux ont été publiés (généralisations à des bases non entières, approches par la logique, par les substitutions...) dont plusieurs sont issus de l'équipe de logique de l'UMH. Je compte faire le point sur ce thème dans mon exposé.

3 Roland HINNION (Bruxelles)

Tree-properties for ordered sets

The notion of "tree" (and "well-pruned tree") of height κ, with θ-finite levels (for $\kappa,\ \theta$ cardinals) can be naturally generalized to become the one of "$\theta - tree$ on a partial order $(E, \leqslant)$"; so the classical "tree-property" (or "ramifiability") inspires several variantsof "tree-properties for orders". The most interesting cases concern directed orders D, have applications to compactness problems [1] and fixed point problems [2] for set-continuous operators, and present interesting links with "large cardinals" [3]; one can show that they cannot be reduced simply to (classical) tree-properties for the main parameters of D, namely the "characteristic cardinal" (the largest δ such that any δ-finite subset of D has an upper bound) and the "cofinality" (the least possible cardinal of a cofinal subset of D). Several "combinatorial criteria" have been established [3, 4], some of which use not necessarily directed orders, motivating investigation of "tree-properties for (partial) orders"; reasonable sufficient conditions [4], necessary conditions [5] and in some cases (as: $\theta = \aleph_0$; E finite; E countable) more "geometric" characterizations appeared [5]. Several very natural questions however are still not completely solved; for example: "ramifiability" for a cartesian product of directed sets, with respect to adequate "ramifiability" of the factors; or "absolute well-pruned ramifiability" (i.e. relative to all well-pruned trees on E) for countable orders.

References

[1] Hinnion, R. *Ramifiable directed sets.* Math. Logic Quarterly **44**, 216–228 (1998).

[2] Dzierzgowski, D.; Esser, O. and Hinnion, R. *Fixed-points of set-continuous operators*. Math. Logic Quarterly **46** (2000)

[3] Esser, O. and Hinnion, R. *Large cardinals and ramifiability for directed sets*. Math. Logic Quarterly **46** (2000)

[4] Esser, O. and Hinnion, R. *Combinatorial criteria for ramifiable ordered sets* (submitted)

[5] Esser, O. and Hinnion, R. *Tree-properties for ordered sets* (submitted)

4 Françoise POINT (Mons)

Groupes satisfaisant une identité et propriété de Milnor

In the first part of the talk we will try to give an overview of some results on groups satisfying an identity. Then, we will give a generalization of the Milnor property (introduced by J. Milnor to show that a finitely generated soluble group with that property is polycyclic) and prove a nilpotency criteria in the class of finitely generated soluble group (extending a result of G. Endimoini).

5 Olivier ESSER (Bruxelles)

The consistency strength of a positive set-theory

The aim of this talk is to present the theory $\mathsf{GPK}^{+}_{\infty}$. This theory has a comprehension scheme for bounded positive formulas and an axiom scheme of *closure* which behaves like a *topological closure*.

Our main result about this theory is that it is mutually interpretable with $\mathsf{KM}+$"*On* has the tree-property"; KM is the Kelley-Morse class-theory; "*On* has the tree-proprety" is the natural translation to the class of ordinals of the corresponding notion for cardinals in ZF.

Another interesting result on $\mathsf{GPK}^{+}_{\infty}$ is that the axiom of choice is inconsistent with it.

6 Arnaud MAES (Mons)

Entrelacs Brunniens et généralisations d'après Stanford

A family of brunnian braids can easily be constructed using a commutator collection process. T. Stanford recently proved that a variation of this process characterize brunnian braids (and generalizations), and gives an algorithm for deciding whether a given braid is brunnian. We present Stanford's results.

7 André PETRY

Faire de l'Analyse non standard sans Logique ?

Comment peut-on faire de l'Analyse non standard sans connaissance spéciale en Logique, en se basant sur une méthode introduite par Keisler.

8 Dirk VAN DALEN (Utrecht)

Foundations of Brouwer's Intuitionism

Already at the time of his dissertation, Brouwer had a more or less coherent philosophical basis not only for his mathematics, but for 'everything'. That is to say, science in general, language, social behaviour, etc. His views on the mathematical universe was determined by the reflections on human mind and consciousness. In particular the non-lawlike nature of the mathematical objects was dictated by his so-called "causal sequences". The properties of the continuum and other objects of mathematical practice were based on an analysis of the underlying sequence structure. A number of consequences will be demonstrated, such as the extremely connected nature of the continuum, and even of the irrationals and similar sets. From the logical point of view, the intuitionistic structures of real life ask for a highly refined analysis. This can be seen already in simple theories such as that of equality.

9 Serge GRIGORIEFF (Paris)
Automates et mots transfinis

Les langages de mots transfinis reconnus par automate ont été introduits par Büchi pour prouver la décidabilité de la théorie monadique des ordinaux $\leqslant \aleph_1$. Nous étudions les relations de mots transfinis. En particulier, nous montrons que l'uniformisation des relations rationnelles est possible pour les langages de mots de longueur $< \omega^n$ mais échoue à partir de l'ordinal ω^ω.

10 Henk BARENDREGT (Nijmegen)

Set theory versus type theory as foundation

A foundation for mathematics is an axiomatic system such that most of mathematics can be formalised in it. This formalisation can be done either in principle or actually. There are reasons why an 'in principle' foundation is useful. Set theory provides such a foundation. There are reasons why an 'actual' foundation is useful. Type theory provides such a foundation.

11 Marco FORTI (Pisa)

Wanted: A strongly comprehensive theory of collections

We present an axiomatization of the primitive notions of *collection* and *set*, stemming from the foundational programme of E. De Giorgi. This axiomatization is intended to capture the most general concept underlying the naive notions of *class* or *aggregate*, as conceived by Frege and Cantor. In our view, these notions try to mediate the somewhat different concepts of "extension of an arbitrary property" and of "content of a finite list". The former is captured in our theory by the notion of *collection*, and the latter by that of *set* ="*small manageable collection"*. Cantor's set theory, later axiomatized by Zermelo, isolates sets as "not too big" collections, which can be freely manipulated. Von Neumann's axiomatization of sets and classes identifies "sethood" and "elementhood", a *Limitation of Size Principle* being the basic criterion for both. We consider all collections, big and small, as *first class objects*, and not merely as a "façon de parler". Hence we introduce the collection *Coll* of all collections and consider various "Gödel Operations" acting on *Coll*: *pairing, difference, cartesian product, etc.* The idea that sets are "small" is embodied in the Axiom of Replacement, while their "simple and controlled internal structure" is axiomatized by assuming that the graph of *membership* between sets and objects is a collection.

Although strong *Comprehension Principles* can be derived, in the usual Bernays' style, we cannot apply consistently to arbitrary collections all the manipulations carried out in ordinary mathematical practice. In particular *union, intersection,* and *cartesian product* of a collection of collections may not exist as collections (and so does the collection of *all subcollections*). These constructions play an important rôle in almost all areas of Mathematics, Logic and Semantics, and it seems appropriate to introduce operations $Un, Int, Cart, Sub\_Col$

that carry out the intended tasks, so as to make substantial use of collections. So far we have only proved the consistency of the weaker axiom:

Un, Int *act on every set of collections and every collection of sets.* $Cart$ *acts on every set of collections.*

We conjecture (at least) that arbitrary unions and intersections are indeed consistent, and we pose the question to the attention of all interested scholars.

12 Elisabeth BOUSCAREN (Paris)

Théorie des modèles et Conjecture de Manin-Mumford (d'après Ehud Hrushovski)

Nous présenterons des applications récentes de la Théorie des Modèles à des questions de Géométrie Diophantienne sur les corps de nombres. Nous indiquerons en particulier comment E.Hrushovski, en utilisant la Théorie des corps algébriquement clos munis d'un automorphisme, donne une nouvelle démonstration de la conjecture de Manin-Mumford, démonstration qui produit de bonnes bornes effectives.

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Institut de Mathématique et d'Informatique
Université de Mons-Hainaut
« Le Pentagone », 6 av. du champ de Mars
7000 Mons, Belgique